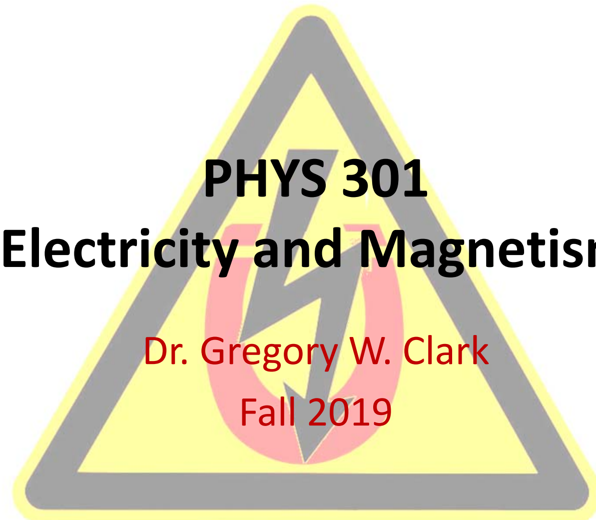
PHYS 301
Electricity and Magnetism



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Today!

- Quiz!!
- Integral vector calculus

Two Theorems:

For Curl-less (irrotational) fields:

$$i)\quad \vec{\nabla}\times\vec{F} = 0$$
$$ii)\quad \int_a^b \vec{F}\cdot d\vec{l} \text{ is path independent}$$
$$iii)\quad \oint \vec{F}\cdot d\vec{l} = 0 \ \forall \text{ closed loops}$$
$$iv)\quad \vec{F} = \vec{\nabla} V$$

EQUIVALENT!

Recall that $\vec{\nabla}\times(\vec{\nabla}f)=0$

For Divergence-less (solenoidal) fields:

$$i)\quad \vec{\nabla}\cdot\vec{F} = 0 \ \forall \text{ space}$$
$$ii)\quad \int \vec{F}\cdot d\vec{a} \text{ is surface independent}$$
$$iii)\quad \oint \vec{F}\cdot d\vec{a} = 0 \ \forall \text{ closed surfaces}$$
$$iv)\quad \vec{F} = \vec{\nabla}\times\vec{A}$$

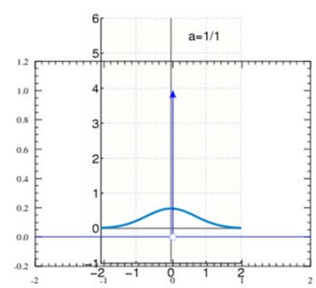
EQUIVALENT!

Recall that $\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right)=0$

The Dirac Delta Function

In 1D: $$\delta(x)=\begin{cases}0, & x\neq 0\\ \infty, & x=0\end{cases}$$

AND $$\int_{-\infty}^{+\infty}\delta(x)\,dx=1$$



More generally,

$$\delta(x-a)=\begin{cases}0, & x\neq a\\ \infty, & x=a\end{cases}$$

and $$\int_{-\infty}^{+\infty}\delta(x-a)\,dx=1$$

$$\therefore \int_{-\infty}^{+\infty} f(x)\delta(x-a)dx = f(a)$$

NOTE:

$$f(x)\delta(x)=f(0)\delta(x)$$
$$\forall \text{ "normal" functions } f(x)$$

$$\therefore \int_{-\infty}^{+\infty} f(x)\delta(x)dx = f(0)\int_{-\infty}^{+\infty}\delta(x)dx = f(0)$$

Limits are actually arbitrary, as long as they include $x = 0$!

THE DIRAC DELTA FUNCTION

In 3D:

$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z)$$

where $\vec{r} \equiv \hat{i}x + \hat{j}y + \hat{k}z$

and $\int_{all\ space} \delta^3(\vec{r})\,d\tau = 1$

$$\therefore \int_{all\ space} f(\vec{r})\delta^3(\vec{r}-\vec{r}_o)\,d\tau = f(\vec{r}_o)$$

So ....

it turns out that

$$\vec{\nabla}\cdot\left(\frac{\vec{\rho}}{\rho^3}\right) = 4\pi\delta^3(\vec{\rho})$$

$$\forall\ \vec{\rho} \equiv \vec{r} - \vec{r}_o$$

including

$\vec{r}_o = 0$

Integral Calculus: Curls

- The **fundamental theorem of curls**:

$$\int_{surface} \left(\vec{\nabla}\times\vec{V}\right)\cdot d\vec{A} = \oint_{line} \vec{V}\cdot d\vec{l}$$

- Also called ***STOKES' THEOREM***

- NOTE:

$$\oint\left(\vec{\nabla}\times\vec{V}\right)\cdot d\vec{A} = 0 \ \forall \text{ closed surfaces}$$

$\int\left(\vec{\nabla}\times\vec{V}\right)\cdot d\vec{A}$ depends only on the boundary; not on the surface!

e.g., soap bubble!